Experiment 10: Circular Motion

Previously we discussed two dimensional projectile motion, but another example of motion in two dimensions is that of circular motion. The simplest case is when the radius and velocity of the moving body are held constant. However, even though the speed is constant, the direction of the velocity is continually changing and the body is, therefore, always undergoing acceleration. Imagine a body, originally at some point on a circle designated by radius r_o and angle θ, with a velocity, $\overrightarrow{v}$, tangential to the circle (see Figure 1). Its position may be written as:

$$\overrightarrow{r} = r_o cos\theta\widehat{i} + r_o sin\theta\widehat{j} \quad (1)$$

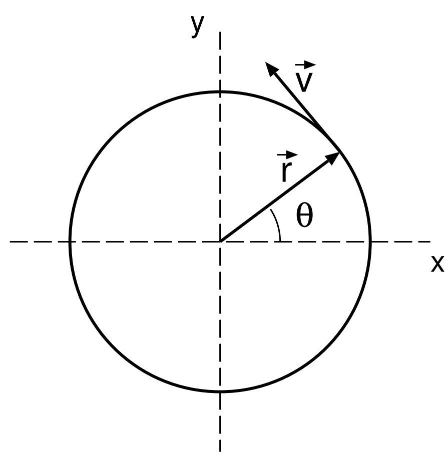

Figure 1: A particle moving at constant speed v in a circular path of radius r.

Since the particle is under uniform circular motion, the angle θ changes with time as:

$$\theta = \omega t \quad (2)$$

where ω is the (constant) angular velocity. Thus, combining Equations 1 and 2, we obtain the time dependence of the body's position

$$\overrightarrow{r} = r_o cos(\omega t)\widehat{i} + r_o sin(\omega t)\widehat{j} \quad (3)$$

The velocity and the acceleration of the body are then obtained by applying Equation 3 with $v = d\vec{r}/dt$ and $a = d\vec{v}/dt$:

$$\overrightarrow{v} = -\omega r_o sin(\omega t)\widehat{i} + \omega r_o cos(\omega t)\widehat{j} \quad (4)$$

$$\overrightarrow{a} = -\omega^2 r_o cos(\omega t)\widehat{i} - \omega^2 r_o sin(\omega t)\widehat{j} \quad (5)$$

By looking at Equations 4 and 5, it is clear that the position, velocity and acceleration vectors all have constant magnitudes as a function of time ($r_o, \omega r_o, \omega^2 r_o$, respectively). The direction of the acceleration is always perpendicular to the direction of the velocity, which is always perpendicular to the direction of the position vector. The acceleration always points towards the center of the circle.

Since the moving body is experiencing an acceleration towards the center of the circle, then there must be a net force in that direction to keep it under acceleration. By applying Newton's second law (Note: The equations deal only with the magnitudes, so no vector notation is needed):

$$F = ma = m\omega^2 r_o = m\frac{v^2}{r_o} \tag{6}$$

This is the equation we will study in this lab.

Experimental Objectives

The rotating platform used in this experiment is shown in Figure 2. There is a spring on the central rotation axis of the system which provides the force required to keep the brass bob in uniform circular motion. You can adjust the spring length and the position of the bob. The position of the bob during rotation can be monitored with the orange marker/indicator as it aligns with the reference slot. The platform is connected to the computer interface through a rotational motion sensor. You will be able to record the angular velocity of the rotation using the computer interface. Your TA will explain how to operate the platform and collect data with the Data Studio software. Besides the platform, you will need a set of different hanging weights.

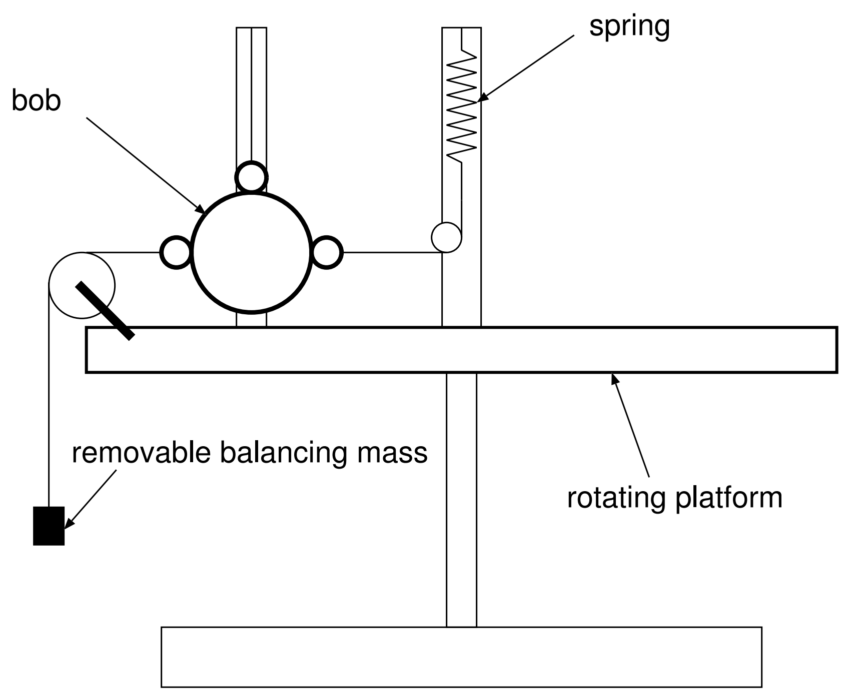

Figure 2: The rotating platform.

- Devise an experimental procedure to verify that the accelerating force is proportional to the square of the angular velocity of rotation. *Hint*: Do not spin the rotating platform with the hanging mass attached. Make a plot of F vs. ω^2 and explain the slope in your lab report.
- Devise an experimental procedure to verify that the force that keeps the bob in uniform circular motion is proportional to the radius of the rotation.