Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART I

Thursday, August 23, 2018

4:00 - 8:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

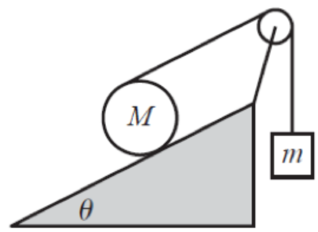
Physical Constants

Speed of light in vacuum	c	=	2.998×10^8 m/s
Planck's constant	h	=	6.626×10^{-34} J$\cdot$ s
	$\hbar$	=	$h/2\pi$
		=	1.055×10^{-34} J$\cdot$s
		=	6.582×10^{-16} eV$\cdot$s
Permeability constant	μ_o	=	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_o}$	=	8.988×10^9 N$\cdot$m^2/C^2
Fine structure constant	α	=	$\frac{e^2}{4\pi\epsilon_o\hbar c}$
		=	$7.30 \times 10^{-3} = \frac{1}{137}$
Gravitational constant	G	=	6.67×10^{-11} m^3/s^2$\cdot$kg
Avogadro's number	N_A	=	6.023×10^{23} mole^{-1}
Boltzmann's constant	k	=	1.381×10^{-23} J/K
		=	8.617×10^{-5} eV/K
kT at room temperature	$k\cdot300$ K	=	0.0258 eV
Universal gas constant	R	=	8.314 J/mole$\cdot$K
Stefan-Boltzmann constant	σ	=	5.67×10^{-8} W/m^2$\cdot$K^4
Electron charge magnitude	e	=	1.602×10^{-19} C
Electron rest mass	m_e	=	9.109×10^{-31} kg
		=	0.5110 MeV/c^2
Neutron rest mass	m_n	=	1.675×10^{-27} kg
		=	939.6 MeV/c^2
Proton rest mass	m_p	=	1.672×10^{-27} kg
		=	938.3 MeV/c^2
Deuteron rest mass	m_d	=	3.343×10^{-27} kg
		=	1875.6 MeV/c^2
Atomic mass unit (C^{12} = 12)	u	=	1.661×10^{-27} kg
		=	931.5 MeV/c^2
Mass of earth	M_E	=	5.98×10^{24} kg
Radius of earth	R_E	=	6.37×10^6 m
Mass of sun	M_S	=	1.99×10^{30} kg
Radius of sun	R_S	=	6.96×10^8 m
Gravitational acceleration at earth's surface	g	=	9.81 m/s^2
Atmospheric pressure		=	1.01×10^5 N/m^2
Radius of earth's orbit		=	1.50×10^{11} m
Radius of moon's orbit		=	3.84×10^8 m

Conversion Factors

1 eV	=	1.602×10^{-19} J	1 J	=	6.242×10^{18} eV
1 Å	=	10^{-10} m	1 Fermi	=	10^{-15} m
1 barn (b)	=	10^{-28} m^2	1 in	=	2.54 cm
0° Celsius	=	273.16 K	1 cal	=	4.19 J

Problem 1: A string wraps around a uniform cylinder of mass M, which rests on a fixed plane. The string passes up over a massless pulley and is connected to a mass m, as shown in the figure. Assume that the cylinder rolls without slipping on the plane and that the string is parallel to the plane. What is the acceleration of the mass m? What is the condition on the ratio M/m for which the cylinder accelerates down the plane?



Problem 2: By any method you choose show that the following transformation is canonical:

$$x = \frac{1}{\alpha}\left(\sqrt{2P_1}\sin Q_1 + P_2\right), \quad p_x = \frac{\alpha}{2}\left(\sqrt{2P_1}\cos Q_1 - Q_2\right),$$
$$y = \frac{1}{\alpha}\left(\sqrt{2P_1}\cos Q_1 + Q_2\right), \quad p_y = -\frac{\alpha}{2}\left(\sqrt{2P_1}\sin Q_1 - P_2\right),$$

where α is some fixed parameter.

Apply this transformation to the problem of a particle of charge q moving in a plane that is perpendicular to a constant magnetic field $\mathbf{B}$. Express the Hamiltonian for this problem in the (Q_i, P_i) coordinates, letting the parameter α take the form

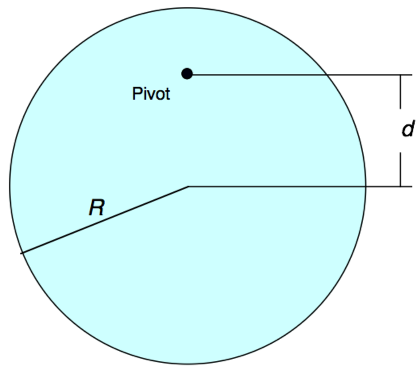
$$\alpha^2 = \frac{qB}{c}.$$

From this Hamiltonian obtain the motion of the particle as a function of time.

Hint: The Hamiltonian $H(x, y, p_x, p_y) = \frac{1}{2m}\left(\mathbf{p} - \frac{q}{c}\mathbf{A}\right)^2$, where $\mathbf{A} = \frac{B}{2}\left(-y\hat{\mathbf{i}} + x\hat{\mathbf{j}}\right)$.

Problem 3: Two identical masses m are constrained to move on a horizontal hoop. Two identical springs with spring constant k connect the masses and wrap around the hoop (see figure). One mass is subject to a driving force $F_d\cos(\omega_d t)$. Find the driven oscillation solution for the motion of the masses.

Problem 4: A physical pendulum consists of a uniform solid disk of mass M and radius R supported in the vertical plane by a pivot located a distance $d < R$ from the center of the disk. The disk is displaced a small angle and released. Derive an expression for the period of the resulting motion.



Problem 5: A tall cylindrical vessel with gaseous nitrogen is located in a uniform gravitational field in which the free-fall acceleration is equal to g. The temperature of the nitrogen varies along its height h so that its density is the same throughout the volume. Find the temperature gradient dT/dh.

Problem 6: Two thermally insulated vessels 1 and 2 are filled with air and connected by a short tube equipped with a valve. The volumes of the vessels, the pressures and temperatures of air in them are known (V_1, p_1, T_1 and V_2, p_2, T_2). Find the air temperature and pressure established after the opening of the valve.

Problem 7: A material contains many two-molecule-binding sites. If neither is occupied, the energy is 0, if one of the two is occupied, the energy is ε, and if both are occupied, the energy is $+3\varepsilon$. Compute the (classical) partition function, the free energy, the energy, and the entropy as a function of inverse temperature β.

Problem 8:

(a) Let 580 nm light be normally incident on a double slit system for which $d = 4800$ nm. How many orders (maxima) will be visible on a screen placed in front of the slits?

(b) If the slit width is $a = 800$ nm, which orders, if any, will be missing?