Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART II

Saturday, August 25, 2018

10:00 AM - 2:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

Physical Constants

Speed of light in vacuum	c	=	2.998×10^{8} m/s
Planck's constant	h	=	6.626×10^{-34} J· s
	$\hbar$	=	$h/2\pi$
		=	1.055×10^{-34} J·s
		=	6.582×10^{-16} eV·s
Permeability constant	μ_o	=	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_o}$	=	8.988×10^{9} N·m^2/C^2
Fine structure constant	α	=	$\frac{e^2}{4\pi\epsilon_o\hbar c}$
		=	$7.30 \times 10^{-3} = \frac{1}{137}$
Gravitational constant	G	=	6.67×10^{-11} m^3/s^2·kg
Avogadro's number	N_A	=	6.023×10^{23} mole^{-1}
Boltzmann's constant	k	=	1.381×10^{-23} J/K
		=	8.617×10^{-5} eV/K
kT at room temperature	k·300 K	=	0.0258 eV
Universal gas constant	R	=	8.314 J/mole·K
Stefan-Boltzmann constant	σ	=	5.67×10^{-8} W/m^2·K^4
Electron charge magnitude	e	=	1.602×10^{-19} C
Electron rest mass	m_e	=	9.109×10^{-31} kg
		=	0.5110 MeV/c^2
Neutron rest mass	m_n	=	1.675×10^{-27} kg
		=	939.6 MeV/c^2
Proton rest mass	m_p	=	1.672×10^{-27} kg
		=	938.3 MeV/c^2
Deuteron rest mass	m_d	=	3.343×10^{-27} kg
		=	1875.6 MeV/c^2
Atomic mass unit ($C^{12} = 12$)	u	=	1.661×10^{-27} kg
		=	931.5 MeV/c^2
Mass of earth	M_E	=	5.98×10^{24} kg
Radius of earth	R_E	=	6.37×10^{6} m
Mass of sun	M_S	=	1.99×10^{30} kg
Radius of sun	R_S	=	6.96×10^{8} m
Gravitational acceleration at earth's surface	g	=	9.81 m/s^2
Atmospheric pressure		=	1.01×10^{5} N/m^2
Radius of earth's orbit		=	1.50×10^{11} m
Radius of moon's orbit		=	3.84×10^{8} m

Conversion Factors

1 eV	=	1.602×10^{-19} J	1 J	=	6.242×10^{18} eV
1 Å	=	10^{-10} m	1 Fermi	=	10^{-15} m
1 barn (b)	=	10^{-28} m^2	1 in	=	2.54 cm
0° Celsius	=	273.16 K	1 cal	=	4.19 J

Problem 1: Superman and the Flash decide to race from Seattle to New York (exactly 4600 km on a map) to see who is faster. Using his super vision, when Superman sees the clock in New York strike 8:00 PM they start off. Superman travels $0.75c$, and the Flash travels at $0.8c$.

(a) What time (to milliseconds) does the clock read when the Flash and Superman each arrive?

(b) How much farther does Superman believe he traveled than the Flash?

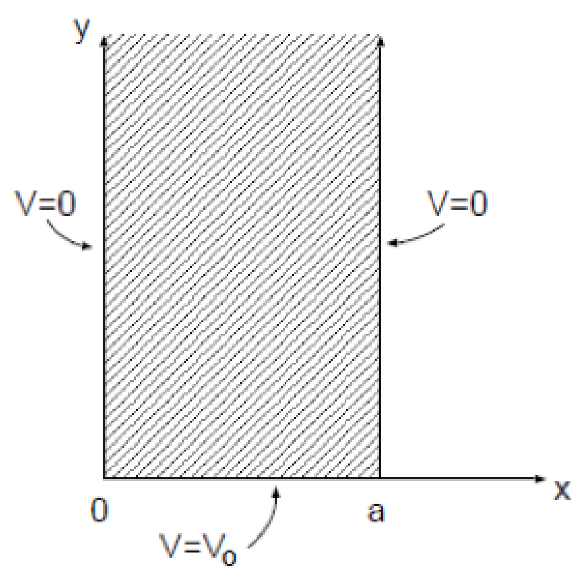
Problem 2: Using the mass of the electron $m_e = 0.511$ MeV/c^2, the fine structure constant $\alpha = 1/137$, and $\hbar c = 197$ MeV·fm, give *BOTH* a symbolic and a numerical solution for the following:

(a) The Compton wavelength λ_C of the electron.

(b) The Bohr radius R_B of a Hydrogen atom.

(c) The speed v of an electron in the lowest Bohr orbit.

Problem 3: Solve Laplace's equation in the region shown in the figure, i.e. for $0 \le x \le a$, $0 \le y \le \infty$. The boundary conditions are $V = 0$ at $x = 0$ and $x = a$, and $V = V_0$ along $y = 0$.



Problem 4: A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What is the smallest magnetic field that would cause the rod to slide?

Problem 5: Show that in a lightly damped RLC circuit ($R \ll \omega L$), the fraction of the energy lost per oscillation is given approximately by $2\pi R/\omega L$.

Problem 6: Consider a two-dimensional rigid rotator in quantum mechanics that describes a molecule rotating in the xy plane. The Hamiltonian for this system is given by

$$H_0 = L^2/2I,$$

where I is the moment of inertia of the rotator, and $L = L_z$ is the orbital angular momentum along the z-axis.

(a) Solve for the eigenfunctions and eigenvalues of H_0.

(b) Suppose a weak perturbation is added to H_0, given by

$$\lambda H' = 2\lambda \cos 2\phi,$$

where ϕ is the angle in the xy plane that the rotator makes with the x-axis ($0 \leq \phi \leq 2\pi$). Calculate the effect of the perturbation on the 1^{st} excited state of the rotator.

Problem 7: For a particle in an infinite square well of width a:

$$V(x) = \begin{cases} 0 & 0 \leq a \\ \infty & \text{otherwise} \end{cases}$$

calculate the first order correction to the **energies and the wavefunctions** due to a perturbing potential

$$W(x) = a\omega_0 \delta(x - a/2),$$

where ω_0 is a real constant with dimension of energy.

Problem 8: An electron can be found in a state Ψ_A near an isolated nucleus A with energy E_A or in a state Ψ_B near an isolated nucleus B with energy E_B. The nuclei are now moved close together so that there is a non-vanishing Hamiltonian matrix element $V = \langle \Psi_B | V | \Psi_A \rangle$ between the two states. Ignoring the direct overlap between the states ($\langle \Psi_B | \Psi_A \rangle \approx 0$), find the new energy levels of the one-electron, two nucleus system. Explain how your result might account for the formation of a stable diatomic molecule.