Experiment 9: Biot-Savart Law with Helmholtz Coil

Introduction

In this lab we will study the magnetic fields of circular current loops using the Biot-Savart law. The Biot-Savart Law states the magnetic field $\mathbf{B}$ from a wire segment length $d\mathbf{s}$, carrying a steady current I is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{I d\mathbf{s} \times \mathbf{r}}{r^2} \tag{1}$$

where μ_0 is the permeability of free space and $\mathbf{r}$ is the displacement vector from the current element $d\mathbf{s}$ to a point P where we wish to evaluate the magnetic field. This equation allows us to calculate the magnetic fields for arbitrary current distributions such as circular or rectangular loops. We will only focus on circular loops in this lab.

The magnitude of the magnetic field B along an axis through the center of a circular loop carrying steady current I (see Fig. 1) can be expressed as

$$B(z) = \frac{\mu_0 I R^2 N}{2(R^2 + z^2)^{3/2}} \tag{2}$$

where R is the radius of the loop and N is the number of turns in the current loop. The current loop has magnetic field lines similar to that of a bar magnet, as shown in Figure 2. The direction of the magnetic field at the center is perpendicular to the plane of the loop, and the direction is given by the *right hand rule* for a current loop: the fingers curl around the loop in the direction of the current, the thumb points in the direction of the magnetic field at the center of the loop. At the center of the loop (N=1), when z=0, the magnetic field is given by
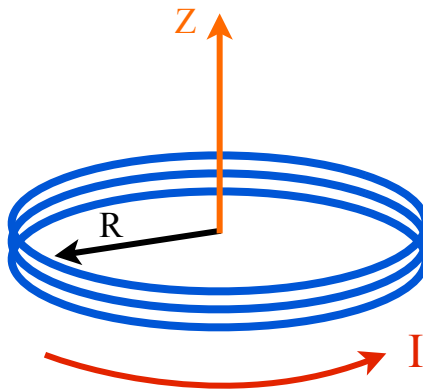
$$B = \frac{\mu_0 I}{2R} \tag{3}$$

Figure 1: A circular loop of wire carrying a current I. The magnetic field direction is given along the positive z-axis.
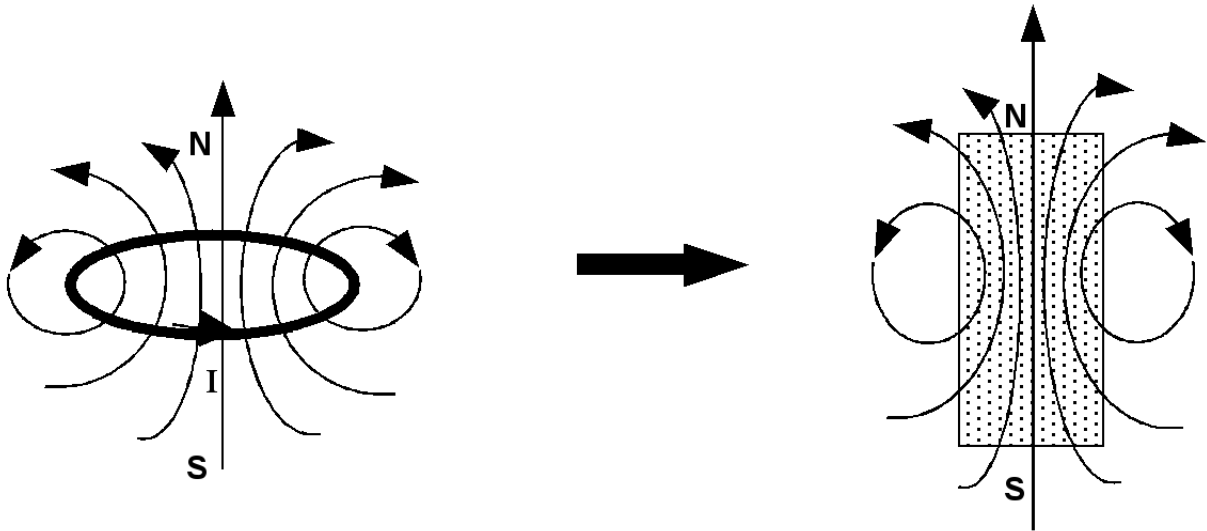
Figure 2: Magnetic field lines for a current loop and a bar magnet. Here the z-axis is along the North-South line.

We can create a stronger and more uniform magnetic field by aligning two identical current loops. A particular configuration that we will be using is known as a *Helmholtz coil*. This consists of two current loops, each with N turns and radius R. The two loops are aligned along their axis and are separated by a distance R, identical to the radius, each carrying equal steady currents in the same direction (see Figure 3). We can use the Biot-Savart law to find the magnetic field at any point along along the axis of the Helmholtz coil by summing the individual magnetic fields of the coils via the superposition principle. It can be shown that the magnetic field at the center of this configuration when z=0 (point O on Fig. 3) is given by

$$B(z = 0) = \frac{8}{5\sqrt{5}} \frac{\mu_0 I N}{R} \tag{4}$$

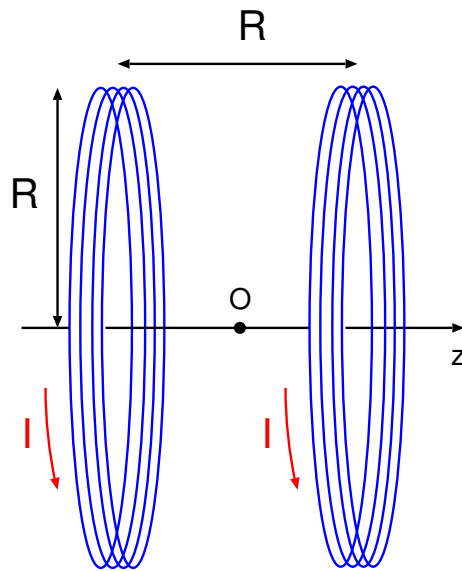where the variables have the same definitions as before.

Figure 3: Helmholtz coil separated by a distance R equal to the radius of the coils.

We will use the *Hall Effect Probe* to measure the magnetic fields. Before using such a device, let's examine how it operates. A brief description will be presented here, but refer to Wikipedia for a more comprehensive explanation. Consider a small slab of semiconductor (shaded rectangle in Figure 4) such as silicon or germanium. Note: all conductors exhibit the Hall effect, but only with semiconductors is the effect large enough to be convenient for practical use. A magnetic field **B** (in this case it is coming out of the page, perpendicular to the current), induces an electric potential - the Hall voltage - across the semiconductor, transverse to the current. Since the Hall voltage is too small to be measured directly with a regular voltmeter, it must be amplified by voltage differential amplifier (triangular object). It takes the voltage difference between the two terminals and amplifies it by a factor of 10 to 20. The magnetic field sensor integrates the semiconductor and amplifiers into a one component design (see Fig. 5a) and can be plugged into port A of the 850 Universal Interface. To set-up the computer interface for the sensor, launch the Capstone program and click on "hardware setup". In the experiment setup window, click analog port A and select "Magnetic Field sensor" . Click on the "digits" option found on the right side, under the displays setting, to see a numerical output of the sensor.
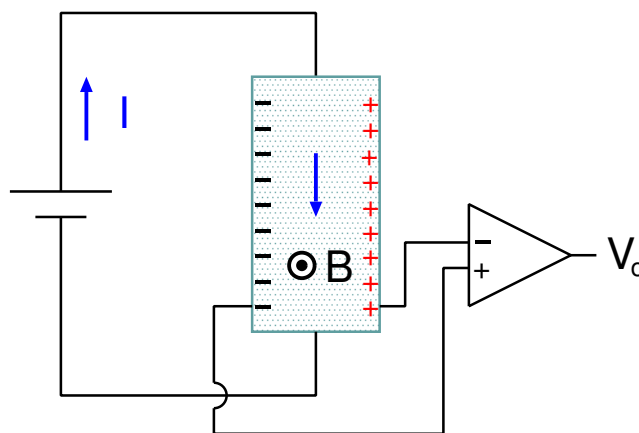

Figure 4: Schematic diagram of a Hall effect probe.

The sensor has two modes of operation: "axial" and "radial" (as seen in Figure 5b). This allows measurements to be taken in the parallel and perpendicular directions, respectively. The probe is sensitive to its orientation with the external magnetic field. The sensing element marked by a dot (see figure 5a) must be pointed in the direction of the field. The sensor has three switch selectable ranges: 1X, 10X, and 100X. Set the range select switch on the sensor to 10X. Make sure to the use Magnetice Field Strength (10X) when you click (Select Measurement) on your digits display, these two setting must agree to display the correct measurement. Please do not use the 100X setting - this can burn out the amplifier. To "zero" the sensor, hold the sensor away from any magnetic source and press the TARE button. The sensor needs to be zeroed after each change of the radial/axial, or range select switch.

Part 1 Procedure - Magnetic Field Strength of a Current Loop:

We will first investigate the magnetic field of a current loop. You are given a coil of wire to create a magnetic field. Record the number of turns N in this coil of wire and its radius R. Connect the coil of wire to a DC power source. Set a current of 1 Amp to flow through the coil using a digital multi-meter in the 20A yellow input to obtain an accurate reading. Measure the magnetic field along the axis of the coil using the Hall probe at 6 different positions, and be sure to take a measurement at the center of the loop. Keep track of the orientation of the probe for each of your readings. Measure the field in front and behind of the coil along the axis until the field drops to near zero. Record in a table the distance z along the axis from the plane of the coil, and the magnitude of the magnetic field B. If you took measurements
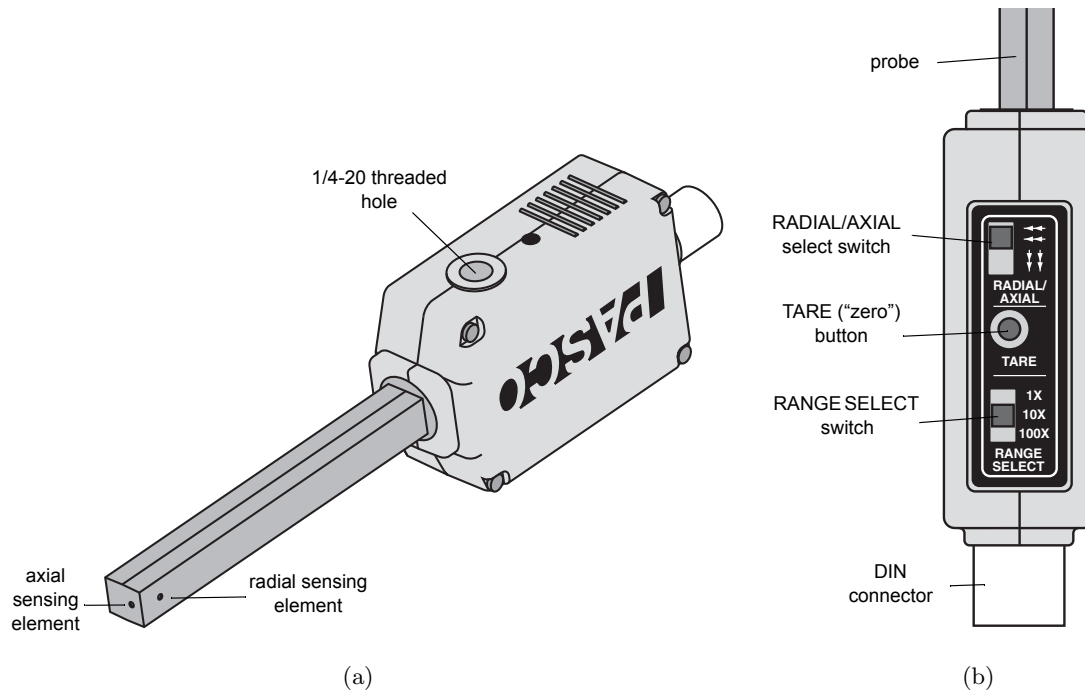
Figure 5: (a) Magnetic field sensor. (b) Top view of probe with setting options.

at identical distances in front and behind the coil, then find the average value of B at that distance.

Part 2 Procedure - Magnetic Field at the Center of a Helmholtz Coil:

Measure the radius of the Helmholtz coil and record the number of turns. Connect the Helmholtz coil to the DC power supply. You may want to use a digital multi-meter in the 20A yellow input to obtain an accurate reading of the current. Using the Hall probe, measure the field at the center of the Helmholtz coil ($z = 0$) for current values from 0.1 A to 0.7 A, in 0.1 Amp increments. Your data table should consist of the different values of I and B(0). *Warning*: The coils might get hot - do not leave them unattended and turn off the current when not in use.

A full lab report is not necessary for this lab. Answer the questions on the following page and turn it in with your signed datasheet.

PHYS 221, Lab 9 Questions

Name: **CWID:**

Write your answers on a separate sheet and attach your signed datasheet when turning it in. You must show all of your work for full credit. Make it clear to me you understand what you're doing. Any graphs or tables should be made via computer software and attached to this handout.

1. Answer the following questions using the data you acquired from Part 1 of this lab:

   (a) Add to your data table(s) a column consisting the value of $1/(R^2 + z^2)^{3/2}$.
   (b) Plot a graph of B(z) versus $1/(R^2 + z^2)^{3/2}$.
   (c) What is the field value at the center of the loop? Compare this value with Equation 3.
   (d) Find the slope of the best-fit line from your graph. From Equation 2, this slope should correspond theoretically to $\mu_0 I R^2 N/2$ (try proving this). Compare the two values.

2. Answer the following questions using the data you acquired from Part 2 of this lab:

   (a) Plot of a graph of B(0) versus I.
   (b) Find the slope of the best-fit line.
   (c) From Equation 2, the slope of this line should correspond to the value of

$$\frac{8}{5\sqrt{5}} \frac{\mu_0 N}{R} \tag{5}$$

   Show how to derive equation 5. Compare the slope of your graph with the value obtained from this expression.