Introduction to the General Physics Laboratories

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1 Course Goals

The goal of the IIT General Physics laboratories is for you to learn to be experimental scientists. For this reason, you will notice that the laboratory manuals are short and do not contain details of the experimental procedures. You will be asked to devise your own experimental procedures and make decisions as to how thoroughly to acquire data during the laboratory session. As the semester progresses, you will develop a good feeling as to how much data to take and how many trials to run for each set of conditions. Your report will suffer if you do not obtain sufficient data to convincingly report your experimental findings.

The Teaching Assistant is present as a resource, she will not tell you *what to do* but guide you to an understanding of the experiment so that you can make the decisions yourself.

The experiments have been designed to relate with principles you learn in the lecture portion of the course. **There is always a connection!** If you apply the analysis tools you have learned in lecture and homework, the experiments will usually become clear and your understanding of the physics will be strengthened.

2 Good Laboratory Practices

Scientists keep laboratory notebooks in which they record everything they do during an experiment. Everything is written in pen, not in pencil and nothing is erased, just crossed out. You must develop good laboratory habits by keeping complete laboratory notebooks. The best kind of notebook is one where pages cannot be accidentally torn out, but any bound notebook is acceptable. Keep all your notes about the lab procedures and data in this notebook **in pen**. You should never write in pencil in a lab notebook since this becomes a permanent record which can be valuable. If, for instance, you discover a new principle which is patentable, you need to prove when you first discovered it and you have to have the ideas witnessed by an external observer. This is the only way you can protect your intellectual property. It does not matter if you make mistakes in the notebook, just scratch it out and move on. The only requirement is that you write complete notes so that you can understand exactly what you did and measured when you return to write the laboratory report later.

It is always best to read the laboratory manual and lay out a tentative experimental procedure before class. You will undoubtedly change the procedure as you learn the quirks of the apparatus but if you have done some advance work, the whole laboratory session will proceed much more smoothly!

The expectation is that you will work in groups of two or three unless there are too few experimental setups and that you will change lab partners for each experiment. Each of you will write a single report ("article"). The format of the report is described in the document entitled *General Physics Guidelines for Laboratory "Articles"*. We have also provided an example "article".

We do not want you to transcribe your raw data into the laboratory report. You must, however, make sure that the Teaching Assistant has ***signed*** the data sheets in your notebook before you leave the laboratory. These sheets ***must*** be attached to the report so that the Teaching Assistant can check your analysis if necessary.

3 Safety

All laboratories have a strict, enforced dress code, ours are no different. If you don't follow the following code, you will not be permitted to enter the laboratory.

1. no shorts;
2. women may wear skirts, but they must be below the knee,

3. no sandals, no open-toed shoes; and
4. absolutely no food or drink allowed in the laboratory.

In addition there will be personal protective equipment (such as safety glasses) required for certain experiments. You will be informed by the Teaching Assistant when these are necessary.

4 Understanding Experimental Error

There are two types of errors that can occur when taking measurements in a laboratory experiment. *Systematic errors* occur when a measuring device is not properly working or is miscalibrated. For example, an experiment may be carried out a number of times, giving an inaccurate result every time. If there is a large, constant variation between the expected and experimental result, then the error is systematic. Systematic errors can be removed by a proper understanding of your experimental equipment and you should always try to do this through careful experimentation and observation.

Even though a piece of equipment may be in good working order, errors can still occur due to inaccurate readings. These types of errors are called random errors. Random errors occur when a measuring device is improperly read. Random errors may cause different results each time an experiment is performed. It is *impossible* to completely eliminate random errors, however, you can minimize their effect on your experimental results by averaging.

As an experimental physicist it is important to account for, and report your errors. This tells the world how certain you are of your results. In all of the physics labs you will perform this semester (and in coming semesters) you will be expected to use basic statistical tools for the analysis: arithmetic mean (average) and standard deviation. The arithmetic mean is defined as:

$$\langle x \rangle = \frac{1}{N}\sum_{n=1}^{N} x_n \tag{1}$$

where N is the total number of measurements, x_n, is the the n^{th} measurement and n is a running index. Averaging a measurement should have the result of compensating for random errors.

The standard deviation, σ, is defined as:

$$\sigma = \sqrt{\frac{1}{N}\sum_{n}(x_n - \langle x \rangle)^2} \tag{2}$$

When you have a measurement that has random errors, the distribution of values for the measurement will fall in what is called a Normal or Gaussian distribution and it looks like a "bell curve". The standard deviation is the parameter which characterizes such a distribution (it describes a width of the "bell curve"). We generally make the assumption that your measurements will follow this behavior (although we do not check it) and use the standard deviation to determine the consistency of an experiment. A large standard deviation means that there was a large spread in the values of the experiment.

When it is not possible to make multiple measurements of a quantity, it is useful to estimate the measurement error. An example of this is when you use a meter stick calibrated in millimeters as the smallest quantity. It is reasonable to assume that any measurement reported to the nearest 0.5 millimeter has an error of no more that 0.2mm.

The errors described above are called *absolute errors* and they have same units as the quantity itself. By convention, we assign a Δ symbol to denote absolute error. For example, the absolute error in a quantity r is called Δr. When determining the error in a quantity which is a function of two or more other measured quantities which have error, it becomes useful to define the *relative error* of the quantity:

$$\delta r = \frac{\Delta r}{r}. \tag{3}$$

Note that the δ symbol is used for such relative errors. When graphing a quantity, it is always important to show the **absolute** error in the quantity using error bars.

5 Propagation of Error

Often it is necessary to determine the error in a quantity which is computed from two or more independently measured variables, each with it's own error. In this case, we need to have a method for propagating the error in the independent variables to the final computed quantity. This is done by using differentials in the following way.

Suppose a quantity z is a function of the independent variables w, x, and y.

$$z = f(w,x,y) \tag{4}$$

The full differential of z is given by:

$$dz = \frac{\partial f}{\partial w}dw + \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy \tag{5}$$

This expression can be rewritten in terms of the absolute errors to show the relationship between the error in z and the errors in x and y, with the condition that errors in the independent variables can never subtract to give a smaller error in the computed quantity. Therefore we have for the absolute error in z,

$$\Delta z = \left|\frac{\partial f}{\partial w}\right|\Delta w + \left|\frac{\partial f}{\partial x}\right|\Delta x + \left|\frac{\partial f}{\partial y}\right|\Delta y \tag{6}$$

and for the relative error,

$$\delta z = \left|\frac{\partial f}{\partial w}\right|\frac{\Delta w}{z} + \left|\frac{\partial f}{\partial x}\right|\frac{\Delta x}{z} + \left|\frac{\partial f}{\partial y}\right|\frac{\Delta y}{z} \tag{7}$$

This can be extended to any number of independent variables. Note that we have assumed that a standard deviation for a quantity measured multiple times is treated exactly as an estimated error, that is, as an absolute error. There is a method of error propagation which can be used more accurately when all measured quantities have true standard deviations. This is called the “quadrature formula” (Equation 9) and will result in a slightly different error propagation (actually smaller than our estimates here!). See the Appendix if you are interested in using this. For the purposes of these laboratories, we usually have a mix of standard deviations and estimated errors and it is proper to use the kind of error analysis developed here.

Let us see how this works for several simple cases.

5.1 Sum of Variables: $z = w + x - y$

In this case

$$\left|\frac{\partial f}{\partial w}\right| = \left|\frac{\partial f}{\partial x}\right| = \left|\frac{\partial f}{\partial y}\right| = 1$$

and therefore

$$\Delta z = \Delta w + \Delta x + \Delta y$$

The relative error is complex.

5.2 Multiplication by a Constant: $z = Cx$

Here

$$\left|\frac{\partial f}{\partial x}\right| = C$$

and

$$\Delta z = C\Delta x$$

$$\delta z = C\frac{\Delta x}{z} = C\frac{\Delta x}{Cx} = \frac{\Delta x}{x} = \delta x$$

5.3 Product of Variables: $z = xy$

The partial derivatives are given by

$$\left|\frac{\partial f}{\partial x}\right| = y$$

$$\left|\frac{\partial f}{\partial y}\right| = x$$

and we have, for the relative error (which is much simpler in this case

$$\delta z = y\frac{\Delta x}{xy} + x\frac{\Delta y}{xy} = \frac{\Delta x}{x} + \frac{\Delta y}{y} = \delta x + \delta y$$

The quotient of variables works out the same way (try it yourself!).

5.4 Variables Raised to a Power: $z = x^3$

This is a generalization of the product

$$\left|\frac{\partial f}{\partial y}\right| = 3x^2$$

$$\delta z = \left(3x^2\right)\frac{\Delta x}{x^3} = 3\frac{\Delta x}{x} = 3\delta x$$

In general, it is possible to develop an error form for an arbitrary function of as many variables necessary with a bit of patience.

6 Significant Digits

The most common mistake students make is to report a number with too many significant digits. The *precision* of your measurement is determined by the number of significant digits and this depends on your measured quantities and their errors. It is nonsense to report a measured value for the acceleration due to gravity as $g = 9.8103 \pm 0.05$. The error would dictate that the value be reported as $g = 9.81 \pm 0.05$. Furthermore, if you measure two quantities $a = 3.251$ and $b = 2.2$, their product can only be reported to the smallest number of significant digits you have measured (in this case 2), $c = 7.2$. Your calculator may produce 10 digits, but they are meaningless unless all your measurements have this many significant digits. **Be careful!**

7 Graphing

Scientists often try to present their results in a graphical form because it communicates much more information than a number or a verbal description. The most common mistake is to present many graphs, resulting in a loss of meaning. Your job, as a scientist, is to determine which graphs are absolutely necessary. Err on the side of too few rather than too many but make sure that you provide enough to make your case in the report. Another important facet of presenting data in the form of graphs is to choose your axes correctly as to extract the maximum information for discussion. An example of this can be found in the sample laboratory "article". Here we have measured the elapsed time for free fall from various heights. We chose to plot each data point rather than an average value for each height (with error bars, of course) and we chose to plot the height from which the ball is dropped versus $\frac{1}{2}t^2$. This was done because we know that $h = \frac{1}{2}gt^2$ for free fall and by using this choice of abscissa, we can obtain a straight line fit to the data. Straight lines are easiest to fit with most spreadsheet programs and can be easily confirmed by eye. The right choices can make for a much better report, ask your Teaching Assistant for advice if you are unsure.

8 Appendix: The Method of Quadratures

In the case where all the independent variables which make up a your computer quantity are measured in multiple trials, it is possible to develop an expression for the standard deviation of the computed quantity in terms of the standard deviation of the variables. We start with the full differential expansion (Equation 5) and treat the quantities dx, dy, and dz as differences (Note that for simplicity, we have dropped the dependence on w but these results can be extended to any number of independent variables). That is, the deviations of individual measurements from the mean, $dx = x_n - \langle x \rangle$. In this context, we can define the standard deviation of the computed quantity as (from Equation 2):

$$\sigma_z = \sqrt{\frac{1}{N}\sum_n (z_n - \langle z \rangle)^2} \tag{8}$$

Substituting Equation 5 for the quantity $z_n - \langle z \rangle$, we obtain:

\sigma^2

$$\sigma_z^2 = \frac{1}{N}\sum_n \left[\frac{\partial f}{\partial x}(x_n - \langle x \rangle) + \frac{\partial f}{\partial y}(y_n - \langle y \rangle)\right]^2$$

We can expand the quantity on the right side of the equation to obtain

$$\sigma_z^2 = \frac{1}{N}\sum_n \left[\left(\frac{\partial f}{\partial x}\right)^2 (x_n - \langle x \rangle)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (y_n - \langle y \rangle)^2 + 2\left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right)(x_n - \langle x \rangle)(y_n - \langle y \rangle)\right]$$

If the independent variables are truly independent, the cross terms such as $\sum_n (x_n - \langle x \rangle)(y_n - \langle y \rangle) = 0$ and we simplify, using the definition of the standard deviation to

$$\sigma_z^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2$$

and the quadrature formula for standard deviations is

$$\sigma_z = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2} \tag{9}$$