Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART I

Thursday, January 11, 2018

4:00 - 8:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

Physical Constants

Speed of light in vacuum	c	=	2.998×10^{8} m/s
Planck's constant	h	=	6.626×10^{-34} J· s
	$\hbar$	=	$h/2\pi$
		=	1.055×10^{-34} J·s
		=	6.582×10^{-16} eV·s
Permeability constant	μ_o	=	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_o}$	=	8.988×10^{9} N·m^2/C^2
Fine structure constant	α	=	$\frac{e^2}{4\pi\epsilon_o \hbar c}$
		=	$7.30 \times 10^{-3} = \frac{1}{137}$
Gravitational constant	G	=	6.67×10^{-11} m^3/s^2·kg
Avogadro's number	N_A	=	6.023×10^{23} mole^{-1}
Boltzmann's constant	k	=	1.381×10^{-23} J/K
		=	8.617×10^{-5} eV/K
kT at room temperature	k·300 K	=	0.0258 eV
Universal gas constant	R	=	8.314 J/mole·K
Stefan-Boltzmann constant	σ	=	5.67×10^{-8} W/m^2·K^4
Electron charge magnitude	e	=	1.602×10^{-19} C
Electron rest mass	m_e	=	9.109×10^{-31} kg
		=	0.5110 MeV/c^2
Neutron rest mass	m_n	=	1.675×10^{-27} kg
		=	939.6 MeV/c^2
Proton rest mass	m_p	=	1.672×10^{-27} kg
		=	938.3 MeV/c^2
Deuteron rest mass	m_d	=	3.343×10^{-27} kg
		=	1875.6 MeV/c^2
Atomic mass unit ($C^{12} = 12$)	u	=	1.661×10^{-27} kg
		=	931.5 MeV/c^2
Mass of earth	M_E	=	5.98×10^{24} kg
Radius of earth	R_E	=	6.37×10^{6} m
Mass of sun	M_S	=	1.99×10^{30} kg
Radius of sun	R_S	=	6.96×10^{8} m
Gravitational acceleration at earth's surface	g	=	9.81 m/s^2
Atmospheric pressure		=	1.01×10^{5} N/m^2
Radius of earth's orbit		=	1.50×10^{11} m
Radius of moon's orbit		=	3.84×10^{8} m

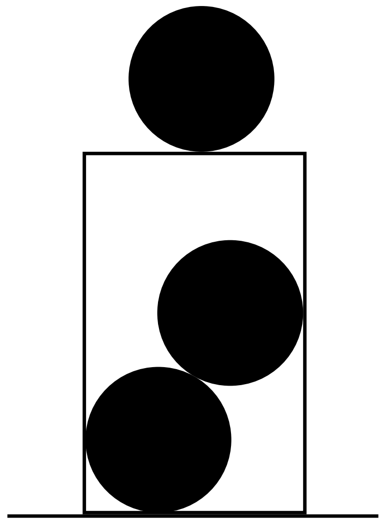
Conversion Factors

1 eV	=	1.602×10^{-19} J	1 J	=	6.242×10^{18} eV
1 Å	=	10^{-10} m	1 Fermi	=	10^{-15} m
1 barn (b)	=	10^{-28} m^2	1 in	=	2.54 cm
0° Celsius	=	273.16 K	1 cal	=	4.19 J

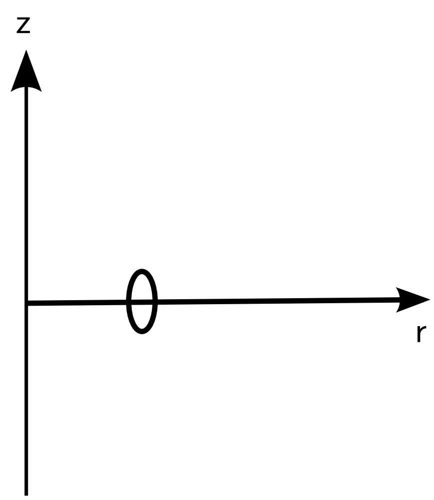
Problem 1: Two kickballs of radius R and mass m are placed in an upside down cylindrical bucket of diameter $3R$, with a third kickball on top (see figure). The ball on top is then removed.

(a) Describe the motion of the bucket in words.

(b) What is the minimum mass of the bucket necessary for the bucket to remain still?

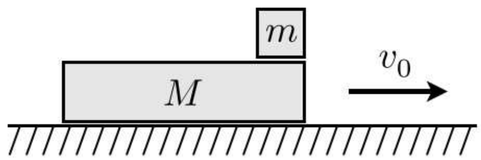


Problem 2: A small ring slides on a wire which rotates uniformly at angular frequency ω in a force-free space. If the wire is straight and perpendicular to the axis of rotation, and the ring starts at the axis of rotation, what is the distance of the ring from the axis as a function of time?



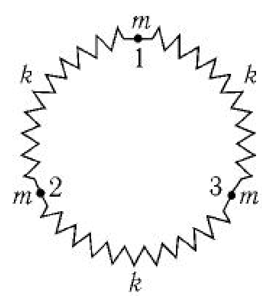
Problem 3:

A board of length L and mass M can slide frictionlessly along a horizontal surface. A small block of mass m initially rests on the board at its right end, as shown in the gure. The coefficient of friction between the block and the board is μ. Starting from rest, the board is set in motion to the right with initial speed v_0. What is the smallest value of v_0 such that the block ends up sliding off the left end of the board? Assume the small block is sufficiently narrow relative to L that its width can be neglected.



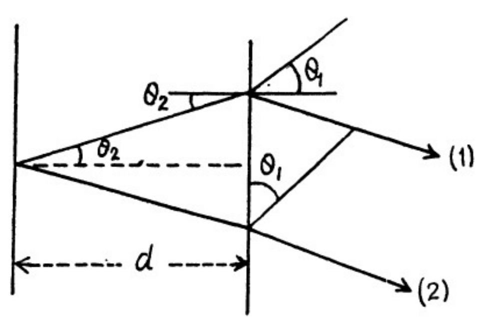
Problem 4:

Three identical point-like masses of mass m are moving on a circle (see the figure). They are connected by identical spring of spring constant k. Determine the eigenfrequencies and fundamental (normal) modes of small oscillations of the system.



Problem 5:

A parallel beam of white light falls on a thin film of the refractive index $n = 1.33$. The incident angle $\theta_1 = 52°$. What should the film thickness be equal to in order to maximaze the yellow ($\lambda = 0.6\,\mu m$) line intensity of the reflected beam?



Problem 6: A thermally insulated cylinder contains either Argon, Methane, or Air at room temperature. The contents are rapidly compressed to a volume 1/2 of the initial volume, and the pressure increases to approximately 2.5 of the initial pressure. Which gas is in the cylinder?

Problem 7:

A system in equilibrium at temperature T of noninteracting spin-one particles of magnetic moment μ is placed in a constant magnetic field $\mathbf{B}$. Derive an expression for the magnetization as a function of temperature.

Problem 8:

A potential energy of molecules in a certain central 3D field depends on a distance r from the field center as $U = ar^2$, where a is a positive constant. The gas temperature is T, the molecules concetration at the center of the field is n_0. Find:

a) The number of molecules at the distance from the center of the field between r and $r + dr$.

b) The most probable distance between a molecule and the center of the field.

c) The fraction of molecules in a spherical layer between r and $r + dr$.

d) How many times the molecules concentration at the center of the field will increase if the temperature increses η times.