MATH 461 – Fourier Series and Boundary-Value Problems

Course Description from Bulletin: Fourier series and integrals. The Laplace, heat, and wave equations: Solution by separation of variables. D'Alembert's solution of the wave equation. Boundary-value problems. (3-0-3)

Enrollment: Required course for AM majors and elective for other majors

Textbook(s): R. Haberman, *Elementary Applied Partial Differential Equations*, 4th ed., Prentice Hall (2004), ISBN 0-13-065243-1.

Other required material:

Prerequisites: MATH 251, MATH 252

Objectives:
1. Students will understand how PDEs (such as the heat and wave equation) model physical phenomena and the basic structure of PDEs and their solutions.
2. Students will learn the separation of variable technique for solving linear second-order PDEs (heat equation, Laplace's equation, wave equation).
3. Students will learn the basics of Fourier series.
4. Students will learn the basic properties of and how to solve regular Sturm-Liouville eigenvalue problems.
5. Students will learn to solve some partial differential equations in more than one space variable.

Lecture schedule: 3 50 minutes (or 2 75 minutes) lectures per week

Course Outline: Hours

1. Heat Equation 6
   a. Derivation in 1D
   b. Different types of boundary conditions
   c. Derivation in 2D/3D
2. Separation of Variables 9
   a. Linearity
   b. Eigenvalues and eigenfunctions
   c. Orthogonality of functions
   d. Separation of variables for heat equation in 1D with various boundary conditions
   e. Separation of variables for Laplace's equation in rectangular and circular domains
   f. Maximum principle for Laplace's equation and well-posedness of Laplace's equation
3. Fourier Series 9
   a. Piecewise smooth and periodic functions
   b. Convergence of Fourier series
   c. Fourier sine and cosine series
   d. Term-by-term differentiation and integration of Fourier series
   e. Complex form of Fourier series

4. Vibrating Strings and Membranes 3
    a. Derivation of 1D wave equation
    b. Boundary conditions
    c. Separation of variables
5. Sturm-Liouville Eigenvalue Problems 12
    a. Motivating examples (e.g., heat equation for non-uniform rod)
    b. Regular Sturm-Liouville problems and their properties (generalized Fourier series, self-adjointness, Green's formula, orthogonality of eigenfunctions, Rayleigh quotient)
    c. Wave equation for non-uniform strings
    d. Rayleigh-Ritz principle and approximation properties
6. Partial Differential Equations in Space 3
    a. Membranes of arbitrary shape
    b. Wave equation in arbitrary regions
    c. Wave equation for circular membranes and Bessel functions

Assessment:	Homework	10-30%
	Quizzes/Tests	20-50%
	Final Exam	30-50%

Syllabus prepared by: Greg Fasshauer and Art Lubin
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