Math 530 – Applied and Computational Algebra

Course Description (Bulletin): Basics of computation with systems of polynomial equations, ideals in polynomial rings; solving systems of equations by Groebner bases; introduction to elimination theory; algebraic varieties in affine n-space; Zariski topology; dimension, degree, their computation and theoretical consequences. (3-0-3) Credit may not be granted for both MATH 431 and MATH 530.

Enrollment: Elective for AM and other majors.

Textbook(s): Cox, Little and O'Shea: Ideals, Varieties and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra.
ISBN 978-1-4419-2257-1 (3rd edition).
Alternate textbook resource: Cox, Little and O'Shea: Using Algebraic Geometry, Graduate Texts in Mathematics, Springer, ISBN 978-0-387-27105-7 (2nd edition).

Other required material: Use of computer algebra system, such as Macaulay2, Singular, CoCoA, or Sage. All are free/open source.

Prerequisites: MATH 332 or MATH 532

Objectives:

1. Students will achieve command of the essentials of computational algebraic geometry and commutative algebra.
2. Students will understand and apply the core definitions and theorems, generating examples as needed, and asking the next natural question.
3. Students will achieve proficiency in constructing proofs, including those using basic polynomial ideal theory, basic algorithms and constructive proofs, basic varieties and existence proofs.
4. Students will achieve proficiency in written and oral communication of proofs and concepts of both pure and applied computational algebraic geometry.
5. Students will become familiar with the major viewpoints and goals of algebraic geometry: ideals, varieties, and algorithms.
6. Students will understand the basic geometric notions of dimension and degree of algebraic sets, the Zariski topology, and its consequences on solving systems of polynomial equations.
7. Students will practice their knowledge of advanced abstract algebra to problems with exercises and applications, through the required use of a computer algebra system, as well as a class project which will consist of reading an extra chapter or a research paper on the topic from the course.

Lecture schedule: 3 50 minute (or 2 75 minute) lectures per week

Course Outline:

Topic	Hours
What is applied algebra? Preliminaries: basic introduction to algebraic structures (fields, rings).	3
Polynomials and affine spaces. Affine varieties and their parametrizations. Dimension and degree.	6
Ideals in the polynomial ring.	3
Polynomials in one variable, and Introduction to algorithms/pseudocode.	3
Monomial orderings and division algorithm in many variables. Dickson's Lemma. The Hilbert basis theorem. The ascending chain condition.	8
Groebner bases and their properties. S-pairs. Buchberger's algorithm and first application of Groebner bases.	6
Elimination and extension theorems. Geometry of elimination. The Zariski topology. Implicitization problem and algorithms for polynomial and rational implicitization. Resultants.	6
Hilbert's NullStellenSatz. Radical ideals, ideal-variety correspondence, radical membership. Primary decomposition.	3

Note: Some of the last three topics may be covered in less depth depending on time constraints. In some semesters, emphasis may be placed on one of the three final topics more so than the other two, in order to cover it in more depth.

Assessment:
Homework 15-30%
Quizzes/discussion/participation 10%
Mid-term exam 20-25%
Final Exam 20-30%
Project 15-25%

Syllabus prepared by: Sonja Petrović, 3/18/2015