MATH 543 – Stochastic Analysis

Course Description from Bulletin: This course will introduce the student to modern finite dimensional stochastic analysis and its applications in finance and insurance. The topics will include: a) an overview of modern theory of stochastic processes, with focus on semimartingales and their characteristics, b) stochastic calculus for semimartingales, including Ito formula and stochastic integration with respect to semimartingales, c) stochastic differential equations (SDE's) driven by semimartingales, with focus on stochastic SDE's driven by Levy processes, d) absolutely continuous changes of measures for semimartingales, e) some selected applications. (3-0-3)

Enrollment: Graduate elective

Textbook(s): Klebaner, Fima C., *Introduction to Stochastic Calculus with Applications,* 2nd ed., Imperial College Press

Other required material:

Prerequisites: MATH 475, or consent of an instructor

Objectives:

1. Students will understand the concept and basic properties of two fundamental stochastic processes in continuous time: Brownian motion and Poisson processes.
2. Students will understand the concept and basic properties of continuous time semi-martingales.
3. Students will understand the concept, properties and use of stochastic exponents.
4. Students will understand the basic tools of stochastic calculus: stochastic integral with respect to a semi-martingale, qudratic variation and predicatble quadratic variation, Ito formula for semi-martingales, etc.
5. Students will understand the concept and use of Girsanov theorem for semi-martingales.

Lecture schedule: 2 75 minute lectures

Course Outline:	Hours
1. Preliminaries from calculus	3
a. Variation of function	
b. Riemann and Riemann-Stieltjes integrals	
c. Differentials and integrals	
d. Other useful stuff	
2. Preliminaries from probability	3
a. Fields and filtrations: discrete model	
b. Continuous model	
c. Lebesgue-Stieltjes integral and expectations	
d. Independence and conditioning	
e. Stochastic processes	
3. Martingales	12
a. Definitions	

b. Basic examples: Brownian motion, Poisson process and related martingales
c. Uniform integrability
d. Martingale convergence
e. Optional stopping
f. Quadratic variation and predictable quadratic variation; martingale representations of Brownian motion
g. Stochastic integrals
h. Localization and local martingales
i. Martingale inequalities
j. Martingale representation
k. Random change of time

4. Semimartingales 15
   a. Definitions and basic examples
   b. Quadratic variation and covariance
   c. Predictable processes
   d. Boob-Meyer decompositions
   e. Stochastic integrals
   f. Ito formula I
   g. Stochastic exponent
   h. Sharp bracket process and compensators
   i. Ito formula II
5. Change of measure and Girsanov theorem 6
   a. Change of measure for random variables
   b. Absolutely continuous probability measures
   c. Girsanov theorem
6. Stochastic Differential Equations 6
   a. Basic concepts
   b. Existence and uniqueness of solutions
   c. Selected properties of solutions
   d. Jump diffusion processes and related IPDEs
   e. Removal of drift
   f. Backward SDEs

Assessment:

	Homework	0-10%
	Quizzes/Tests	45-50%
	Graduate Project	0-10%
	Final Exam	45-50%

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