MATH 544 – Stochastic Dynamics

Course Description from Bulletin: This is an introductory course in mathematical modeling by stochastic differential equations. It is especially appropriate for graduate students who would like to use stochastic methods in their research, or to learn these methods for long term career development. Topics include random variables, mean and variance, Brownian motion, stochastic integration and Ito calculus, stochastic differential equations, random dynamics, numerical simulation, and applications to scientific, engineering and financial problems.

Enrollment: Graduate elective

Textbook(s): B. Oksendal, *Stochastic Differential Equations: An introduction with Applications*, 6th ed., Springer-Verlag

Other required material: Matlab

Prerequisites: MATH 474 or MATH 475 or equivalent

Objectives:

1. Students will learn Brownian motion (Wiener process) and stochastic calculus.
2. Students will learn stochastic differential equations in the context of mathematical modeling.
3. Students will learn basic concepts in stochastic dynamics, i.e., stochastic continuity and stability, convergence in probability and convergence in mean-square, analytical and numerical solution methods, distribution and moments of solutions.
4. Students will learn basic techniques and methods for analyzing stochastic dynamics, i.e., Fokker-Planck equation, exit problem, Lyapunov exponents and ergodic theory, invariant manifold reduction, and estimation of the impact of noise.
5. Students will learn how to simulate stochastic dynamics in Matlab.

Lecture schedule: 2 75 minute lectures

Course Outline:	Hours
1. Basic topics	16
a. Random variables and Brownian motion	2
b. Stochastic continuity and mean-square convergence	2
c. Stochastic integrals	2
d. Stochastic differential equations	2
e. Numerical simulation of stochastic differential equations via Matlab	2
f. Ito's formula	2
g. Fokker-Planck equation	2
h. Exit/first passage problem	2
2. Additional topics	26
a. Dynamical behavior of nonlinear systems under random influences	3
b. Dynamical systems approach for stochastic differential equations	3

c. Liapunov exponents and ergodic theory 3
d. Stochastic bifurcation 3
e. Phenomena induced by noise 3
f. Impact of noise 3
g. Invariant manifold reduction of random systems 3
h. Macroscopic modeling of random system 3
i. Slow invariant manifolds and non-equilibrium dynamics 3

Assessment:	Homework	10-30%
	Computer Programs/Project	10-20%
	Quizzes/Tests	20-50%
	Final Exam	30-50%

Syllabus prepared by: Jeffrey Duan and Tom Bielecki
Date: 12/17/05