Math 545 – Stochastic Partial Differential Equations

Course Description from Bulletin: This course introduces various methods for understanding solutions and dynamical behaviors of stochastic partial differential equations arising from mathematical modeling in science and engineering and other areas. It is designed for graduate students who would like to use stochastic methods in their research or to learn such methods for long term career development. Topics include: Random variables, Brownian motion and stochastic calculus in Hilbert spaces; Stochastic heat equation; Stochastic wave equation; Analytical and approximation techniques; Stochastic numerical simulations via Matlab; Dynamical impact of noises; Stochastic flows and cocycles; Invariant measures, Lyapunov exponents and ergodicity; and applications to engineering and science and other areas.

Enrollment: Elective for AM and other majors.

Textbook(s): G. Da Prato & J. Zabczyk: *Stochastic Equations in Infinite Dimensions*, Cambridge University Press, 1992.
J. Duan: Lecture Notes on ``*Dynamical Systems Methods for Stochastic Partial Differential Equations*'', 2006.

Other required material: Matlab

Prerequisites: MATH 543 or 544, or consent of the instructor

Objectives:

1. Students will learn Brownian motion (Wiener process) and stochastic calculus in Hilbert spaces.
2. Students will learn stochastic partial differential equations in the context of mathematical modeling.
3. Students will learn basic concepts in stochastic dynamics, i.e., stochastic continuity and stability, convergence in probability and convergence in mean-square, distribution and moments of solutions.
4. Students will learn basic techniques and methods for analyzing stochastic dynamics, i.e., Lyapunov exponents and ergodic theory, invariant manifold reduction, macroscopic reduction, and estimation of the impact of noise.
5. Students will learn how to simulate solutions of stochastic partial differential equations in Matlab.
6. Students will improve their presentation and writing skills.

Lecture schedule: 2 75 minutes (or 3 50 minutes) lectures per week

Course Outline:

		Hours
a.	Random variables and Brownian motion in Hilbert spaces	2
b.	Stochastic calculus in Hilbert spaces	4
c.	Ito's formula	4
d.	Stochastic heat/parabolic and wave/hyperbolic equations	8
e.	Stability, distribution and moments of solutions	4

f. Numerical simulation of stochastic differential equations via Matlab 4
g. Dynamical systems approach for stochastic partial differential equations 4
h. Liapunov exponents and ergodic theory 2
i. Stochastic bifurcation 2
j. Noise-induced phenomena 2
k. Invariant manifold reduction of random systems 4
l. Macroscopic modeling of random systems 2

Assessment:	Homework	10-30%
	Computer Programs/Project	10-20%
	Quizzes/Tests	20-50%
	Final Exam	30-50%

Syllabus prepared by: Jeffrey Duan
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