Math 577 – Computational Mathematics I

Course Description from Bulletin: Fundamentals of matrix theory; least squares problems; computer arithmetic, conditioning and stability; direct and iterative methods for linear systems; and eigenvalue problems. Credit may not be granted for both MATH 577 and MATH 477. (3-0-3)

Enrollment: Elective for AM and other majors.

Textbook(s): Lloyd N. Trefethen and D. Bau, Numerical Linear Algebra, SIAM (1997), ISBN 0-89871-361-7.
D. Kincaid and W. Cheney, Numerical Analysis: Mathematics of Scientific Computing, 3rd Ed, Brooks/Cole (2002), ISBN 0-534-38905-8.

Other required material: Matlab

Prerequisites: An undergraduate numerical course such as MATH 350, or consent of the instructor

Objectives:

- 1. Students will understand the basic matrix factorization methods for solving systems of linear equations and linear least squares problems and their derivations.
- 2. Students will understand basic computer arithmetic and the concepts of conditioning and stability of a numerical method.
- 3. Students will understand the basic numerical methods for computing eigenvalues and their derivation.
- 4. Students will understand the basic iterative methods for solving systems of linear equations and their derivation.
- 5. Students will learn how to implement and use these numerical methods in Matlab (or another similar software package).
- 6. Students will improve their problem solving skills in computational mathematics.
- 7. Students will improve their presentation and writing skills.

Lecture schedule: 3 50 minutes (or 2 75 minutes) lectures per week

| Course Outline: | Hours |
|---------------------------------------|-------|
| 1. Fundamentals | 5 |
| a. Matrix-vector multiplication | |
| b. Orthogonal vectors and matrices | |
| c. Norms | |
| d. Computer arithmetic | |
| 2. Singular Value Decomposition | 3 |
| 3. QR Factorization and Least Squares | 8 |
| a. Projectors | |
| b. QR factorization | |
| c. Gram-Schmidt orthogonalization | |
| | |

d. Householder triangularization

| e. | Least squares problems | | |
|-------------|--------------------------------------|----------|---|
| 4. Conditi | oning and Stability | | 5 |
| a. | Conditioning and condition numbers | 8 | |
| b. | Stability | | |
| 5. System | s of Equations | | 5 |
| a. | Gaussian elimination | | |
| b. | Cholesky factorization | | |
| 6. Eigenv | alues | | 8 |
| a. | Overview of eigenvalue algorithms | | |
| b. | Reduction to Hessenberg or tridiagon | nal form | |
| с. | Rayleigh quotient, inverse iteration | | |
| d. | QR Algorithm without and with shif | ts | |
| e. | Computing the SVD | | |
| 7. Iterativ | e Methods | | 8 |
| a. | Overview of iterative methods | | |
| b. | Arnoldi iteration | | |
| с. | GMRES | | |
| d. | Conjugate gradients | | |
| e. | Preconditioning | | |
| Assessment: | Homework | 10-30% | |
| | Computer Programs/Project | 10-20% | |
| | Quizzes/Tests | 20-50% | |
| | Final Exam | 30-50% | |

Syllabus prepared by: Greg Fasshauer and Xiaofan Li **Date**: Oct.19, 2005, updated Jan.24, 2008