# MATH 579 – Complexity of Numerical Problems

**Course Description from Bulletin:** This course is concerned with a branch of complexity theory. It studies the intrinsic complexity of numerical problems, that is, the minimum effort required for the approximate solution of a given problem up to a given error. Based on a precise theoretical foundation, lower bounds are established, i.e. bounds that hold for all algorithms. We also study the optimality of known algorithms, and describe ways to develop new algorithms if the known ones are not optimal. (3-0-3).

**Enrollment:** Elective for AM and CS graduate students

**Textbook(s)/References:** H. Niederreiter, *Random Number Generation and Quasi-Monte Carlo Methods*, CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, 1992.
E. Novak, *Deterministic and stochastic error bounds in numerical analysis*, Springer-Verlag, Lectures Notes in Mathematics Vol. 1349, Berlin, 1988.
K. Ritter, *Average-Case Analysis of Numerical Problems*, Lecture Notes in Mathematics Vol. 1733, Springer-Verlag, Berlin, 2000.
I. H. Sloan and S. Joe, *Lattice Methods for Multiple Integration*, Oxford University Press, Oxford, 1994
V. N. Temlyakov, *Approximation of Periodic Functions*, Computational Mathematics and Analysis Series, Nova Science Publishers, Inc., Commack, NY, 1993.
J. F. Traub, G. W. Wasilkowski and H. Wozniakowski, *Information-Based Complexity*, Academic Press, Boston, 1988.
J. F. Traub and A. G. Werschulz, *Complexity and Information*, Cambridge University Press, Cambridge, 1998
G. Wahba, *Spline Models for Observational Data*, CBMS-NSF Regional Conference Series in Applied Mathematics, Vol. 59, SIAM, Philadelphia, 1990

**Other required material:** None

**Prerequisites:** MATH 471 Numerical Methods

**Objectives:**
This subject teaches students how to find optimal numerical algorithms. Specifically

1. Students will learn how to precisely define a numerical problem.
2. Students will learn how to determine a lower bound on the computational complexity of a problem.
3. Students will learn how to analyze the error of an algorithm and determine if it is optimal.
4. Students will learn how to apply these techniques to search, integration and approximation problems.

**Lecture schedule:** 3 50 minute (or 2 75 minute) lectures per week

**Course Outline:** Hours

| Topic | Hours |
|---|---|
| 1. General Theory | 6 |
| a. What is The Complexity of Numerical Problems? | |
| b. Basic Notions | |
| c. Complexity of the Search Problem | |
| 2. Integration | 12 |
| a. Reproducing kernel Hilbert spaces and error bounds | |
| b. Lower bounds on the complexity | |
| c. Product rules versus quasi-Monte Carlo Rules | |
| 3. Tractability | 6 |
| a. How error bounds depend on the dimension | |
| b. What problems and algorithms yield weak dependence on the dimension | |
| 4. Randomized and Average Case Complexity | 6 |
| 5. Complexity and Tractability for Approximation | 6 |
| 6. Selected topics | 6 |

**Assessment**:

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|---|---|
| Homework | 25–30% |
| Presentation | 25–30% |
| Final Exam | 40–50% |

**Syllabus prepared by**: Fred Hickernell and Greg Fasshauer
**Date**: 01/12/06