Math 581 – Finite Elements

Course Description from Bulletin: Various elements, error estimates, methods for solving systems of linear equations including multigrid, discontinuous Galerkin methods. Applications. (3-0-3)

Enrollment: Elective for AM and other majors.

Textbook(s):

1. Claes Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method*, Dover (2009), ISBN 048646900X, 978-0486469003.
2. C. Shu, *Discontinuous Galerkin Methods: General Approach and Stability*, Lecture Notes

Other required material: Matlab

Prerequisites: Undergraduate courses in numerical methods (such as Math 350) and in partial differential equations (such as Math 489), or consent of the instructor.

Objectives:

1. Students will understand how to discretize elliptic PDEs with the finite element method (FEM).
2. Students will understand the concepts of finite element spaces and error estimates.
3. Students will understand the basics of the multigrid method.
4. Students will understand the basics of the DG method.
5. Students will learn how to implement and use these numerical methods in Matlab (or another similar software package).
6. Students will improve their problem solving skills in computational mathematics.
7. Students will improve their presentation and writing skills.

Lecture schedule: 2 75-minute lectures per week

Course Outline: Lectures

1. Introduction to FEM for elliptic problems 8
    a. Variational formulation of a 1D model
    b. FEM for the model problem with piecewise linear functions
    c. An error estimate for the model problem
    d. FEM for the Poisson equation
    e. Some math concepts: Hilbert Spaces
    f. Geometric interpretation of FEM
    g. Natural and essential boundary conditions
    h. Remarks on FEM software
2. Discretization and approximation theory for FEM 5
    a. Regularity requirement
    b. Some examples of finite elements
    c. Interpolation with piecewise polynomials in 2D
    d. Discretization and error estimates for FEM for elliptic problems

    e. Adaptive methods
3. Some Applications in elliptic problems 2
    a. The elasticity problem
    b. Stokes' problem
4. Methods for solving systems of linear equations 8
    a. Direct methods
    b. Iterative methods: overview
    c. Conjugate gradient method
    d. Preconditioning
    e. Multigrid methods
5. Discontinuous Galerkin (DG) methods 9
    a. Time discretization
    b. DG method for conservation laws (hyperbolic equations)
    c. DG method for convection-diffusion equations
    d. DG method for PDEs with higher-order derivatives

Assessment:	Homework	20-40%
	Computer Programs/Project	20-40%
	Quizzes/Tests	10-40%
	Final Exam	20-40%

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