Math 590 – Meshfree Methods

Course Description from Bulletin: Fundamentals of multivariate meshfree radial basis function and moving least squares methods; applications to multivariate interpolation and least squares approximation problems; applications to the numerical solution of partial differential equations; implementation in Matlab. (3-0-3)

Enrollment: Advanced graduate course in computational mathematics. Also suitable for graduate students in other (engineering) departments.

Textbook(s): G. Fasshauer, *Meshfree Approximation Methods with Matlab*, Lecture Notes, IIT, 2005.

Other required material:

Prerequisites: Some exposure to computational mathematics and advanced analysis, consent of the instructor

Objectives:

1. Students will learn the definitions and understand the key concepts of multivariate scattered data approximation with radial basis functions and moving least squares methods.
2. Students will learn direct and iterative algorithms to solve multivariate interpolation and least squares approximation problems.
3. Students will learn how to apply these methods to the solution of partial differential equations.
4. Students will learn how to implement and use these algorithms in Matlab.
5. Students will improve their problem solving skills in computational mathematics.
6. Students will improve their presentation and writing skills.

Lecture schedule: 2 75 minutes (or 3 50 minutes) lectures per week

Course Outline:	Hours
1. Multivariate Scattered Data Interpolation	12
a. Radial Basis Function Interpolation in Matlab	
b. Positive Definite and Completely Monotone Functions	
c. Scattered Data Interpolation with Polynomial Precision	
d. Conditionally Positive Definite Functions	
e. Compactly Supported Radial Basis Functions	
2. Reproducing Kernel Hilbert Spaces	3
a. Error Bounds	
3. Stability and Trade-Off Principles	3
4. Optimality of Radial Basis Function Interpolation	1
5. Least Squares Approximation	5
a. Least Squares RBF Approximation and Smoothing of Noisy Data	
b. Moving Least Squares Approximation	
c. Approximate MLS Approximation	

6. Fast Algorithms 5
    a. Fast Fourier Transform for Non-uniform Data
    b. Partition of Unity Methods and Approximation of Point Cloud Data in R^3
    c. Residual Iteration
    d. Adaptive Iteration
    e. Fast Multipole-type Algorithms
7. Preconditioning Techniques 3
8. Generalized Hermite Interpolation 2
9. Solution of Partial Differential Equations 8
    a. Elliptic PDEs via RBF Collocation
    b. RBF-Pseudospectral Methods for Time-Dependent PDEs

Assessment:	Homework	10-30%
	Computer Programs/Project	10-20%
	Quizzes/Tests	20-50%
	Final Exam	30-50%

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