Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART II

Saturday, August 24, 2019

1:00–5:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

Physical Constants

Speed of light in vacuum	c	$=$	2.998×10^{8} m/s
Planck's constant	h	$=$	6.626×10^{-34} J$\cdot$ s
	$\hbar$	$=$	$h/2\pi$
		$=$	1.055×10^{-34} J$\cdot$s
		$=$	6.582×10^{-16} eV$\cdot$s
Permeability constant	$\mu_{\rm o}$	$=$	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_{\rm o}}$	$=$	8.988×10^{9} N$\cdot$m^2/C^2
Fine structure constant	α	$=$	$\frac{e^2}{4\pi\epsilon_{\rm o}\hbar c}$
		$=$	$7.30 \times 10^{-3} = \frac{1}{137}$
Gravitational constant	G	$=$	6.67×10^{-11} m^3/s$^2\cdot$kg
Avogadro's number	N_A	$=$	6.023×10^{23} mole^{-1}
Boltzmann's constant	k	$=$	1.381×10^{-23} J/K
		$=$	8.617×10^{-5} eV/K
kT at room temperature	$k\cdot 300$ K	$=$	0.0258 eV
Universal gas constant	R	$=$	8.314 J/mole$\cdot$K
Stefan-Boltzmann constant	σ	$=$	5.67×10^{-8} W/m$^2\cdot$K^4
Electron charge magnitude	e	$=$	1.602×10^{-19} C
Electron rest mass	m_e	$=$	9.109×10^{-31} kg
		$=$	0.5110 MeV/c^2
Neutron rest mass	m_n	$=$	1.675×10^{-27} kg
		$=$	939.6 MeV/c^2
Proton rest mass	m_p	$=$	1.672×10^{-27} kg
		$=$	938.3 MeV/c^2
Deuteron rest mass	m_d	$=$	3.343×10^{-27} kg
		$=$	1875.6 MeV/c^2
Atomic mass unit ($C^{12} = 12$)	u	$=$	1.661×10^{-27} kg
		$=$	931.5 MeV/c^2
Mass of earth	$M_{\rm E}$	$=$	5.98×10^{24} kg
Radius of earth	$R_{\rm E}$	$=$	6.37×10^{6} m
Mass of sun	$M_{\rm S}$	$=$	1.99×10^{30} kg
Radius of sun	$R_{\rm S}$	$=$	6.96×10^{8} m
Gravitational acceleration at earth's surface	g	$=$	9.81 m/s^2
Atmospheric pressure		$=$	1.01×10^{5} N/m^2
Radius of earth's orbit		$=$	1.50×10^{11} m
Radius of moon's orbit		$=$	3.84×10^{8} m

Conversion Factors

1 eV	$=$	1.602×10^{-19} J	1 J	$=$	6.242×10^{18} eV
1 Å	$=$	10^{-10} m	1 Fermi	$=$	10^{-15} m
1 barn (b)	$=$	10^{-28} m^2	1 in	$=$	2.54 cm
0° Celsius	$=$	273.16 K	1 cal	$=$	4.19 J

Problem 1: A long straight piece of copper (Cu) wire with a cross sectional radius a, and electrical resistance R, is bent in half to form two long parallel wires of length l whose centers are a distance d apart.

(a) Neglecting any flux within the wire itself, find the self inductance L of the wire in this configuration.

(b) Assume a battery of emf ε is connected to the wire at time $t = 0$. Derive an expression for the current $i(t)$.

Problem 2: A point charge q is located at distances a and b from two perpendicular conducting half-planes, both at zero potential. Calculate the force acting on the charge q.

Problem 3: A surface of a non-conducting infinitely thin spherical shell of radius R is charged with the surface charge density $\sigma(\theta) = \sigma_0 \cos\theta$. Find the electrostatic potential inside and outside the shell.

Hint: Recall the Laplace operator in spherical system of coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2},$$

and necessary boundary conditions.

Problem 4: Consider N non-interacting particles confined in a one-dimensional infinite well:

$$V(x) = \begin{cases} \infty, & x \le 0,\, x \ge a \\ 0, & 0 < x < a \end{cases}$$

Find a ground state energy of these N particles if they are:

(a) Fermions with a spin of $s = 1/2$.

A useful sum formula:

$$\sum_{n=1}^{k} n^2 = \frac{k(k+1)(2k+1)}{6}.$$

(b) Bosons with a spin of $s = 0$.

(c) Bosons with a spin of $s = 1$.

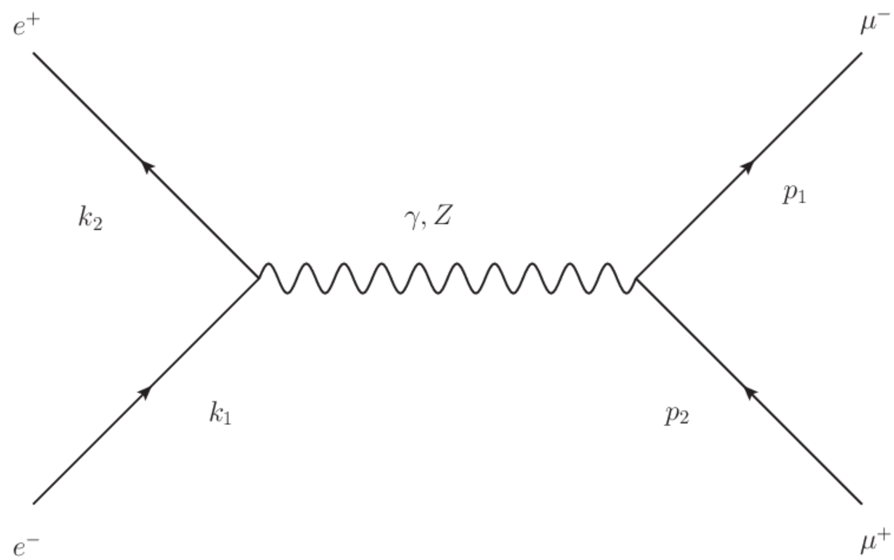
Problem 5: Find eigenvalues and eigenfunctions of a spin operator $\hat{\sigma}_n = \hat{\boldsymbol{\sigma}} \cdot \mathbf{n}$, where $\mathbf{n}$ is a unit vector with components $\mathbf{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ and $\hat{\boldsymbol{\sigma}}$ are Pauli matrices.

Problem 6: Using WKB (quasi-classical) approximation, estimate a transmission coefficient through a potential barrier:

$$U(x) = \begin{cases} 0 & x > |x_0| \\ \\ U_0 - \frac{1}{2}m\omega^2 x^2 & x \leq |x_0| \end{cases}$$

for a particle of mass m and energy $0 < E < U_0$. To define x_0, use a continuity requirement for the potential energy $U(x)$.

Problem 7:



Let us consider a reaction $e^+ + e^- \to \mu^+ + \mu^-$. Calculate a threshold of the reaction (a minimum allowed energy of an electron beam that is necessary to let a reaction go on) for two historically important cases.

(a) A collider. Electron and positron beams are going head on with equal energies.

(b) A fixed target. An electron beam collides with positrons at rest.

(c) If you neglected the electron mass, give an oder-of-magnitude estimate for your error.

Electron and muon rest energies are equal to $m_e c^2 = 0.511\,\text{MeV}$ and $m_\mu c^2 = 105.7\,\text{MeV}$ respectively.

Problem 8: A special intergalactic game involves one player sending a square box of side 1 m off in some direction with large rockets. The box is open in the front and back. Player two tries to get a 2 m long dart into the box. The first player wins if he can close both ends of the box on the dart or the second player misses. The second player wins if she has any part of the dart sticking out of the box when player one tries to close both ends. If the first player sends his box out at speed $0.7c$, what is the maximum speed the second player can send her dart and win?