Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART I

Thursday, January 17, 2019

4:00–8:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

Physical Constants

Speed of light in vacuum	c	$=$	2.998×10^{8} m/s
Planck's constant	h	$=$	6.626×10^{-34} J· s
	$\hbar$	$=$	$h/2\pi$
		$=$	1.055×10^{-34} J·s
		$=$	6.582×10^{-16} eV·s
Permeability constant	μ_o	$=$	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_o}$	$=$	8.988×10^{9} N·m^2/C^2
Fine structure constant	α	$=$	$\frac{e^2}{4\pi\epsilon_o\hbar c}$
		$=$	$7.30 \times 10^{-3} = \frac{1}{137}$
Gravitational constant	G	$=$	6.67×10^{-11} m^3/s^2·kg
Avogadro's number	N_A	$=$	6.023×10^{23} mole^{-1}
Boltzmann's constant	k	$=$	1.381×10^{-23} J/K
		$=$	8.617×10^{-5} eV/K
kT at room temperature	$k{\cdot}300$ K	$=$	0.0258 eV
Universal gas constant	R	$=$	8.314 J/mole·K
Stefan-Boltzmann constant	σ	$=$	5.67×10^{-8} W/m^2·K^4
Electron charge magnitude	e	$=$	1.602×10^{-19} C
Electron rest mass	m_e	$=$	9.109×10^{-31} kg
		$=$	0.5110 MeV/c^2
Neutron rest mass	m_n	$=$	1.675×10^{-27} kg
		$=$	939.6 MeV/c^2
Proton rest mass	m_p	$=$	1.672×10^{-27} kg
		$=$	938.3 MeV/c^2
Deuteron rest mass	m_d	$=$	3.343×10^{-27} kg
		$=$	1875.6 MeV/c^2
Atomic mass unit ($C^{12} = 12$)	u	$=$	1.661×10^{-27} kg
		$=$	931.5 MeV/c^2
Mass of earth	M_E	$=$	5.98×10^{24} kg
Radius of earth	R_E	$=$	6.37×10^{6} m
Mass of sun	M_S	$=$	1.99×10^{30} kg
Radius of sun	R_S	$=$	6.96×10^{8} m
Gravitational acceleration at earth's surface	g	$=$	9.81 m/s^2
Atmospheric pressure		$=$	1.01×10^{5} N/m^2
Radius of earth's orbit		$=$	1.50×10^{11} m
Radius of moon's orbit		$=$	3.84×10^{8} m

Conversion Factors

1 eV	$=$	1.602×10^{-19} J	1 J	$=$	6.242×10^{18} eV
1 Å	$=$	10^{-10} m	1 Fermi	$=$	10^{-15} m
1 barn (b)	$=$	10^{-28} m^2	1 in	$=$	2.54 cm
0° Celsius	$=$	273.16 K	1 cal	$=$	4.19 J

Problem 1: A mass m falls through the air under the force of gravity (gravitational acceleration g), subject to quadratic drag force bv^2, where v is the vertical speed (take as positive downward) and b is a drag parameter. It starts from rest and the motion is entirely vertical.

(a) What is the terminal velocity v_t? Express your answer in terms of m, g, and b.

(b) Write down the differential equation of motion and solve for $v(t)$. Express your result as $v(t) = v_t \tanh(t/\tau)$ and calculate the time constant τ in terms of m, g, and b.

(c) Sketch a graph of the speed as a function of time.

Problem 2: The moon's surface gravitational acceleration is about $1/6$ that of the earth. The moon is thought to have an average composition similar to that of the earth's mantle. If we estimate the density of the moon as $2/3$ that of the earth, use these estimated values to calculate:

(a) the ratio of the radii of earth and moon;

(b) the ratio of the masses of earth and moon.

Explain your reasoning.

The lunar tidal force is the vector difference between the moon's gravitational force that is felt by an object on the earth's surface and the corresponding force if the object were placed at the center of the earth. The earth–moon distance is about $240,000$ miles and the earth's radius is about 4000 miles.

(c) Using the information above, approximately calculate the magnitude of the tidal force on the points on earth's surface that are nearest and farthest from the moon.

(d) From this and the observation that the surface of the ocean is approximately an equipotential surface, estimate the change in elevation of the water surface from lunar tides. Explain your reasoning.

Problem 3: Three equal masses m are constrained to move along the x axis. The masses on the ends are connected to the middle mass by springs of equal spring constant k, but the system is not attached to anything else.

(a) Write down the Lagrangian for the system.

(b) Determine the equations of motion using the Euler–Lagrange equation.

(c) Determine the normal frequencies and modes (eigenvalues and eigenvectors) and explain why they make sense.

Problem 4: A projectile of mass m is fired at an angle θ above the horizontal, with an initial velocity v_0. At the highest point of the trajectory, the projectile explodes into two fragments of equal mass. One of the fragments falls vertically with zero initial speed, following the explosion.

(a) How far from the point of firing does the other fragment strike the level terrain?

(b) How much energy was released during the explosion? (Assume that the energy loss in the form of heat and sound may be ignored.)

Problem 5: When a particle with spin 1/2 and magnetic moment μ is placed into a magnetic field $\mathbf{B}$, its energy level is split into $-\mu B$ and μB. Suppose a system consisting of N such particles is in a magnetic field $\mathbf{B}$ and is kept at temperature T. Find the free energy, internal energy, entropy, specific heat, and the total magnetic moment $M = -\dfrac{\partial F}{\partial B}$.

Problem 6: A pressure which a gas exerts on the walls of a vessel can be regarded as the time average of the impulses which the gas molecules impart on the wall when colliding with and recoiling from it. From this point of view, calculate the pressure p and show that

$$p = \frac{2}{3} n \bar{\epsilon},$$

where n is the average number of molecules per unit volume, and $\bar{\epsilon}$ is the average kinetic energy per molecule.

Problem 7: Find an efficiency of a cycle consisting of two isochoric and two adiabatic lines, if the volume of the ideal gas changes by a factor of $n = 10$ within the cycle. The working substance is nitrogen.

Problem 8: A flat plate of thickness d with refractive index n_1 is placed inside a medium with refractive index $n_1 < n_2$. A ray of light from the point S reaches the plate at an angle α_1. What is the lateral displacement BC of the refracted ray that passes through the plate?