Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART II

Saturday, January 19, 2019

1:00–5:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

Physical Constants

Speed of light in vacuum	c	$=$	2.998×10^8 m/s
Planck's constant	h	$=$	6.626×10^{-34} J· s
	$\hbar$	$=$	$h/2\pi$
		$=$	1.055×10^{-34} J·s
		$=$	6.582×10^{-16} eV·s
Permeability constant	μ_o	$=$	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_o}$	$=$	8.988×10^9 N·m^2/C^2
Fine structure constant	α	$=$	$\frac{e^2}{4\pi\epsilon_o \hbar c}$
		$=$	$7.30 \times 10^{-3} = \frac{1}{137}$
Gravitational constant	G	$=$	6.67×10^{-11} m^3/s^2·kg
Avogadro's number	N_A	$=$	6.023×10^{23} mole^{-1}
Boltzmann's constant	k	$=$	1.381×10^{-23} J/K
		$=$	8.617×10^{-5} eV/K
kT at room temperature	k·300 K	$=$	0.0258 eV
Universal gas constant	R	$=$	8.314 J/mole·K
Stefan-Boltzmann constant	σ	$=$	5.67×10^{-8} W/m^2·K^4
Electron charge magnitude	e	$=$	1.602×10^{-19} C
Electron rest mass	m_e	$=$	9.109×10^{-31} kg
		$=$	0.5110 MeV/c^2
Neutron rest mass	m_n	$=$	1.675×10^{-27} kg
		$=$	939.6 MeV/c^2
Proton rest mass	m_p	$=$	1.672×10^{-27} kg
		$=$	938.3 MeV/c^2
Deuteron rest mass	m_d	$=$	3.343×10^{-27} kg
		$=$	1875.6 MeV/c^2
Atomic mass unit ($C^{12} = 12$)	u	$=$	1.661×10^{-27} kg
		$=$	931.5 MeV/c^2
Mass of earth	M_E	$=$	5.98×10^{24} kg
Radius of earth	R_E	$=$	6.37×10^6 m
Mass of sun	M_S	$=$	1.99×10^{30} kg
Radius of sun	R_S	$=$	6.96×10^8 m
Gravitational acceleration at earth's surface	g	$=$	9.81 m/s^2
Atmospheric pressure		$=$	1.01×10^5 N/m^2
Radius of earth's orbit		$=$	1.50×10^{11} m
Radius of moon's orbit		$=$	3.84×10^8 m

Conversion Factors

1 eV	$=$	1.602×10^{-19} J	1 J	$=$	6.242×10^{18} eV
1 Å	$=$	10^{-10} m	1 Fermi	$=$	10^{-15} m
1 barn (b)	$=$	10^{-28} m^2	1 in	$=$	2.54 cm
0° Celsius	$=$	273.16 K	1 cal	$=$	4.19 J

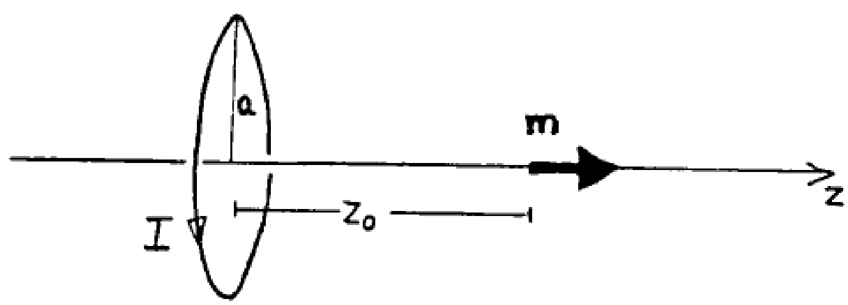
Problem 1: A particle of charge q and mass m is placed at rest at $t = 0$ in perpendicular, constant, electric and magnetic fields $\mathbf{E} = E\hat{\mathbf{z}}$ and $\mathbf{B} = B\hat{\mathbf{x}}$. Calculate the trajectory of the particle and describe it.

Problem 2: A particle of mass M and magnetic dipole moment $\mathbf{m}$ is placed on the axis of a circular current loop of radius a and current I (which is kept fixed), at a distance z_0 from the center of the loop. $\mathbf{m}$ is aligned in the direction of the loop field. (z_0 is not necessarily much greater or smaller than a.)

(a) What is the force of attraction between the loop and $\mathbf{m}$?

(b) When $\mathbf{m}$ is released, it moves toward the center of the loop. What is its kinetic energy when it arrives there? (Assume that $\mathbf{m}$ is constrained to the z axis.)

(c) If the particle is originally placed at the center of the loop, what is the frequency of small oscillation about this position for motion along the z axis?



Problem 3: A plasma generated inside a long hollow cylinder of radius R has the following charge distribution:

$$\rho(r) = \frac{\rho_0}{[1 + (r/a)^2]^2},$$

where r is the distance to the center and ρ_0 and a are constants. Determine the electric field everywhere.

Problem 4: Calculate the transmission and reflection coefficients of a particle having total energy E at the potential barrier given by

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 \leq x \leq a \\ 0 & x > a, \end{cases}$$

for $E > V_0$ case. Under what condition does the barrier turn out to be 100% transparent?

Problem 5: A plane rigid rotor having a moment of inertia I and an electric dipole moment $\mathbf{d}$ is placed in a homogeneous electric field $\mathbf{E}$. By considering the electric field as a perturbation, determine the first non-vanishing correction to the energy levels of the rotor. Hint: you will need to go beyond the first perturbation order.

Problem 6: An electron is placed in a uniform magnetic field $\mathbf{B}$ that is pointing in the z-direction. Initially the state of the electron is

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|\chi_+\rangle + |\chi_-\rangle\right),$$

where $|\chi_\pm\rangle$ are the states in which the z-components of spin of the electron $\pm\hbar/2$.

(a) Given that the magnetic moment of the electron is $\boldsymbol{\mu} = \gamma\mathbf{S}$, find the wavefunction of the electron as a function of time t.

(b) What is the smallest positive value of t such that if the x-component of the electron's spin is measured there is zero probability of getting the value $+\hbar/2$?

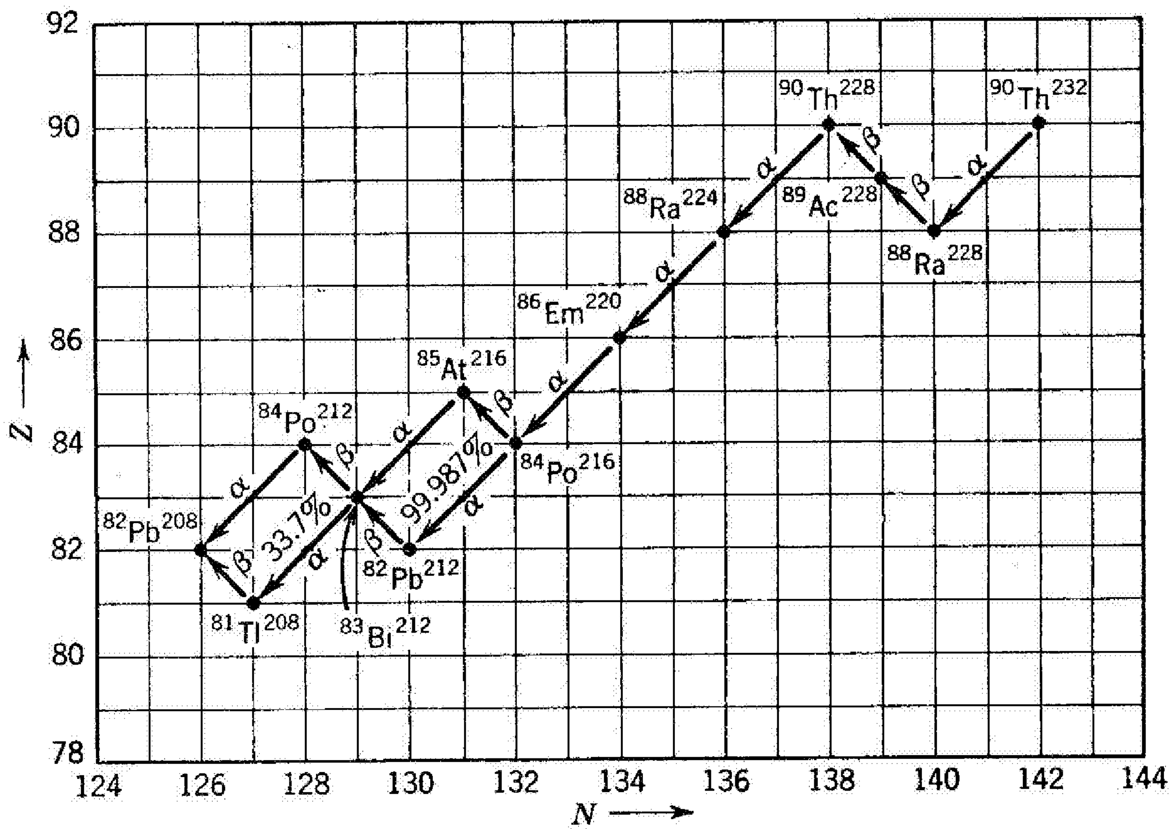
Problem 7: A muon accelerator sends beams of positive and negative muons in opposite directions around a racetrack-shaped storage ring of circumference 6 km. The energy of each beam is 200 GeV. The mass and lifetime of a muon are $m_\mu = 105.7$ MeV/c^2 and $\tau_\mu = 2.197$ μs, respectively.

(a) How many circuits can the muons make before 63% have decayed?

(b) The muons in the μ^- beam each decay to an electron and 2 neutrinos ($\mu^- \to e^-\bar{\nu}_e\nu_\mu$). Some fraction of the neutrinos pass through the Earth and enter a detector that can identify electrons, muons, and their antiparticles. Assuming the muon neutrinos oscillated into either tau neutrinos or electron neutrinos, explain how one could experimentally determine the probability the muons oscillated to each.

(c) At one point along the ring the surviving muons are brought into collision to produce pairs of top quarks ($t\bar{t}$). What is the magnitude of momentum of the outgoing top quarks (in GeV/c) if $m_t = 175$ GeV/c^2?

Problem 8: $^{90}Th^{232}$ decays through a sequence of other isotopes to $^{82}Pb^{208}$. The half-life of $^{90}Th^{232}$ is 1.4×10^{10} yr. A 5 kg sample thorium ore is found to contain 1 kg of $^{82}Pb^{208}$.



(a) Assuming all of the $^{82}Pb^{208}$ originated in the decay of $^{90}Th^{232}$, and none was lost, calculate the age of the rock.

(b) Six α particles are produced in each decay chain from $^{90}Th^{232}$ to $^{82}Pb^{208}$. Assuming a negligible fraction of α particles could escape from the rock, how much helium should be in the rock?

(c) The first daughter in the series $^{88}Ra^{228}$ decays with a half-life of 5.7 years to $^{89}Ac^{228}$. Calculate how much $^{88}Ra^{228}$ should be in the rock.